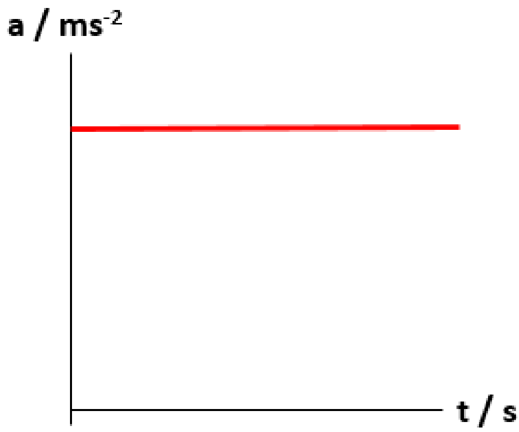
SUVAT Equations

- **3.1.2:** Derive and apply the SUVAT equations of motion.

[redacted]

SUVAT equations are a branch of mechanics that are based on the assumption of constant acceleration. While there are many occasions that this assumption is not valid, there are many occasions this it is and SUVAT equations give a simple strategy to solve problems.



If acceleration is constant, we can draw an acceleration vs. time graph as shown above. The key principle that we are going to be using is that acceleration is the rate of change of velocity or alternatively that can be expressed as the acceleration is the gradient of a velocity vs. time graph.

a = $\frac{dv}{dt}$

⇨ **dv = a dt**

So if an object experiences constant acceleration a for a period of time t, the change in velocity: Δv = at. This is the same as the area under the acceleration vs. time graph from the starting point t = 0 to the time t.

If we call the initial velocity u and the final velocity v

⇨ **Δv = v – u**
⇨ **v – u = at**
⇨ **v = u + at**

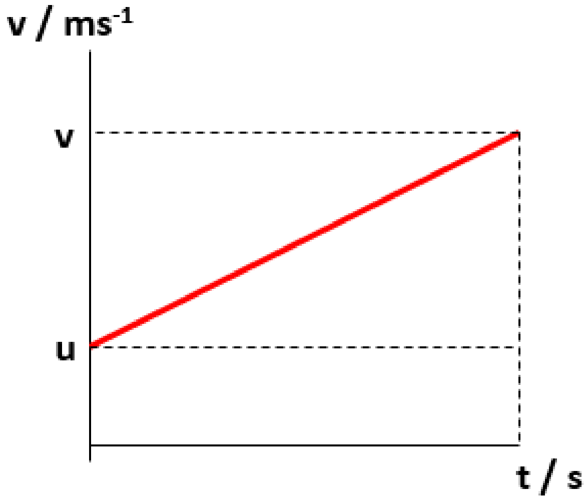
Finding the area under a graph is the same as using the mathematical function integration:

⇨ v – u = $\int_0^t a\,dt$ = at – 0 = at

⇨ **v = u + at** [1]

Again, this integration method only works if a is a constant, if a changes with time, $\int_0^t a\,dt \neq at$

Velocity is defined as the rate of change of displacement or velocity is the gradient of a displacement vs. time graph therefore displacement is the area under a velocity vs. time graph from t = 0 to the time t



The trapezium formed can be split into a rectangle and a triangle so that the total area is calculated:

s = ut + ½ t (v-u)

⇨ **s = ut + ½ vt – ½ ut**

⇨ **s = ½ vt + ½ ut**

⇨ s = $\frac{u+v}{2}t$ [2]

Equation 3:

Substituting [1] into [2]:

s = $\frac{u+u+at}{2}t$

⇨ s = $\frac{2u+at}{2}t$

⇨ **s = ut + ½ at^2** [3]

Doing the same method using Calculus:

v = $\frac{ds}{dt}$

⇨ **ds = v dt**

⇨ **ds = (u + at) dt**

If we say the object starts from an initial displacement s=0 and moves to a displacement s

⇨ **s = (u+at) dt**

⇨ s = $\int_0^t u + at\ dt$

⇨ **s = ut + ½ at^2** [3]

Equation 4:

Re-arranging [1]

⇨ $t = \frac{v - u}{a}$

Substitute re-arrangement into [2]

⇨ $s = \frac{u+v}{2} . \frac{v - u}{a}$

⇨ $2as = v^2 – u^2$

⇨ $v^2 = u^2 + 2as$ **[4]**

<u>Doing the same method using Calculus:</u>

$a = \frac{dv}{dt} = \frac{dv}{ds} . \frac{ds}{dt} = v \frac{dv}{ds}$

⇨ $\int_0^s a ds = \int_u^v v dv$

⇨ $as = ½ (v^2 – u^2)$

⇨ $v^2 = u^2 + 2as$ **[4]**

However the equations are derived, whether using calculus or graph or substitution, the assumption that acceleration is a constant is always made and so SUVAT equations are only appropriate in this condition.

SUVAT and Freefall:

An object is dropped from an initial height of 2.50m. Calculate:

s = -2.5m
u = 0ms^{-1}
v = ?
a = -9.81 ms^{-2}
t = ?

(i) The time taken to hit the ground:

s = ut + ½ at^2

⇨ **s = ½ at^2**

⇨ **t =** $\sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \text{ x } -2.50}{-9.81}}$ **= 0.713922… = <u>0.714s</u> *(3.s.f)***

(ii) The velocity at which the object will hit the ground.

v^2 = u^2 + 2as

⇨ **v =** $\sqrt{2as} = \sqrt{2 \text{ x } -9.81 \text{ x } -2.50}$ **= 7.003571… = <u>7.00 ms^{-1}</u> *(3.s.f)***

(iii) If instead the object is initially thrown upwards at 2ms^{-1}, calculate the new final velocity.

v^2 = u^2 + 2as

⇨ **v =** $\sqrt{u^2 + 2as} = \sqrt{+2^2 + 2 \text{ x } -9.81 \text{ x } -2.50}$ **= 7.283543… = <u>7.3 ms^{-1}</u> *(2.s.f)***

(iv) Verify your answer to (iii) using Conservation of Energy.

GPE_i = mgh_i = 2.5mg

KE_i = ½ mv_i^2 = 2m

⇨ **Total Energy = m (2.5g + 2)**

GPE_f = 0

⇨ **KE_f = m (2.5g + 2)**

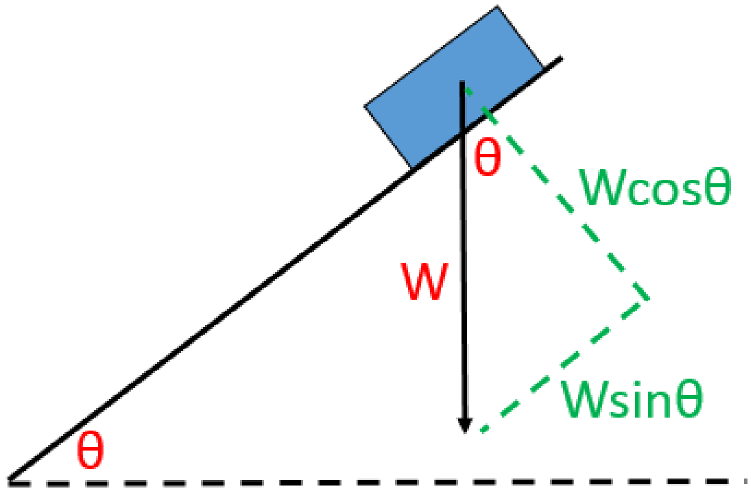
⇨ **½ mv_f^2 = m (2.5g + 2)**

⇨ **v_f^2 = 5g + 4**

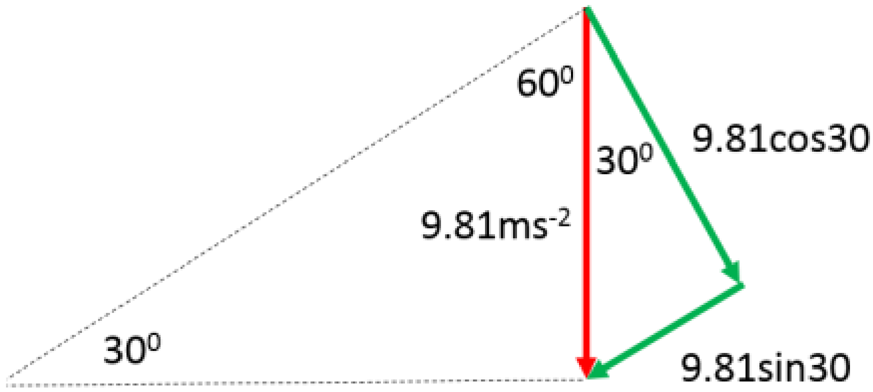
⇨ **v_f = 7.283543… = <u>7.3 ms^{-1}</u> *(2.s.f)***

So you can see that Conservation of Mechanical Energy predicts the same final velocity as SUVAT so SUVAT does not violate the Principle of Conservation of Energy (Always a positive thing!)

SUVAT and Motion on a Slope:



(i) If the slope is at an angle of 30^0, **calculate** the component of acceleration parallel to the slope.



$a_{parallel}$ = 9.81sin30 = 4.905 = <u>4.9 ms^{-1}</u> *(2.s.f)*

(ii) Calculate the velocity of the objects when it has travelled 3m down the slope.

s = 3m

u = 0ms^{-1}

v = ?

a = 4.905ms^{-2}

t = ?

$v^2 = u^2 + 2as$

⇨ **$v = \sqrt{2as} = \sqrt{2 \times 4.905 \times 3}$ = 5.4249… = <u>5.4ms^{-1}</u>** *(2.s.f)*

(iii) Calculate the time taken to travel 3m

$s = ut + ½ at^2$

⇨ **$s = ½ at^2$**

⇨ **$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 3}{4.905}}$ = 1.106… = <u>1.1s</u>** *(2.s.f)*